Coherent voltage oscillations in small normal tunnel junctions and the crossover to the incoherent regime

by Yuval Gefen

We discuss the possibility of charge oscillations in a normal tunnel junction, driven by an external current source (I_{ex}), in the coherent limit. In that limit the dephasing time t_ϕ is larger than the period $t_p \equiv e/I_{ex}$. This behavior is modified when t_ϕ decreases.

1. Introduction

When discussing quantum effects in mesoscopic (typically submicron) systems [1], one should account for several important aspects of the problem. Quantum interference is important on length scales shorter than ℓ_ϕ (the phase-breaking length); fluctuations (both quantum and thermodynamic) may become appreciable, as the thermodynamic limit is not yet approached; the discreteness of charge carriers may also lead to novel effects.

The latter have been emphasized in recent discussions of Bloch oscillations in small current-driven Josephson junctions. These oscillations, first suggested by Widom et al. [2], were later discussed by Likharev and Zorin [3] and by others [4–8]. One considers a small capacitance junction which is driven by a *classical* ideal current source. This source may continuously inject charge into one of the electrodes of the junction. Charge transfer across the junction may take place in discrete quanta of $2e$ or e. Minimizing the instantaneous energy of the system, this gives rise to voltage oscillations at frequency $2e/I_{ex}$ (and e/I_{ex}), where I_{ex} is the externally injected dc current. There is some experimental evidence of interesting charging effects that occur in such systems [9–11], but Bloch oscillations have not been observed so far.

The analysis of this effect involves a few questions, some of which are not yet satisfactorily settled. One issue is the use of a washboard potential versus a time-dependent vector potential to describe an externally driven system not in

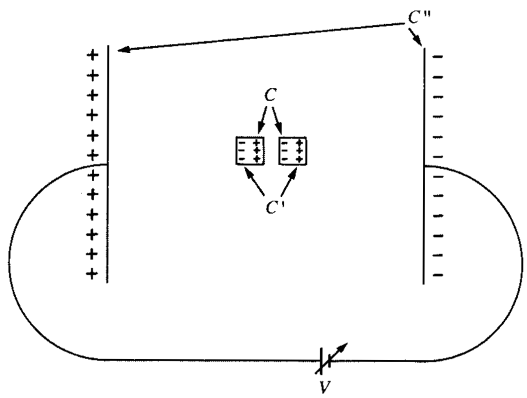

Figure 1

Model current source and small capacitance tunnel junction.

equilibrium. Setting θ as the phase difference of the macroscopic wave function across the Josephson junction, should we identify θ with $\theta + 2\pi$ [12]?* We do not discuss these problems here, but rather focus on similar oscillations that occur in normal tunnel junctions. Ben-Jacob and Gefen [4] suggested that the existence of a superconducting state is not a necessary requirement for the existence of charge oscillations, and, in fact, they should be observed in a small normal tunnel junction as well. A few works that followed treated some aspects of this problem [5, 13–16]. However, all these works have assumed fast thermal equilibration in the electrodes of the junction and have neglected interference effects that may occur during the tunneling (see below).

Thouless and Gefen have recently considered this problem, in the limit of large t_ϕ ("the coherent limit"), and found that the behavior of the junction in that limit is significantly different from the "incoherent" limit considered previously [17]. Here the main results are summarized, emphasizing some of the assumptions made in our analysis and commenting on the conditions needed to observe the charge oscillations. There are several other effects pertinent to these systems, including nonequilibrium noise and nonlinear response; these are not discussed in this paper.

2. Definition of the problem

We consider a normal tunnel junction driven by an ideal dc current source. There are several setups that can be used to model a current source; none of them is a satisfactory microscopic model. The setup we use is shown in **Figure 1**. It consists of two small metallic electrodes embedded in an electrostatic field generated by a large ("classical") capacitor. Such a configuration was previously suggested by Büttiker [8]. This field is varied adiabatically by slowly charging up the large capacitor C''. We assume that the charging of C'' is continuous and uniform in time. This assumption raises some difficulties: The charging current supplied by the battery V consists, of course, of discrete charge carriers. It therefore cannot be uniformly continuous. However, we note that E, the bare electrostatic field in C'' (in the absence of the small junction) is related to the external charge Q'' by

$$E = Q''/C''L, \tag{1}$$

where L is the distance between the two plates of C''. For macroscopic values of the above quantities, the fluctuations in E can be made small [18]. The polarization of the small systems tries now to follow the electrostatic field E. This can be accomplished by occasionally transferring electrons across the junction. We are interested in the dynamics of this process.

At this point let us consider the instantaneous (adiabatic) energy of the system. As we increase V (or Q'', the total charge on C'') linearly in time, the energy stored in the capacitor C'' (and C') increases (there is a volume factor involved). We subtract these energies and refer to the energy of the capacitor C (plus excitations of the electron gas in the electrodes) as the energy of the system.

The internal energy of a noninteracting degenerate electron gas is given by [19]

$$E = E_0 + \left(\frac{\pi}{3}\right)^{2/3} \frac{mk^2}{\hbar^2} NT^2\left(\frac{\vartheta}{N}\right)^{2/3}, \tag{2}$$

where m is the electron mass, N is the number of electrons in the system (i.e., the electrodes), ϑ is the volume, and T is the temperature. E_0 is the internal energy at $T = 0$. When C is not charged, the energy levels of the system form a quasicontinuous spectrum with energy spacing ΔE that satisfies $\log \Delta E(E) \sim -\sqrt{E}$ [19]. As C is being charged adiabatically, the energy of the system increases proportionally to Q^2, hence proportionally to t^2 (t is the time). This is the case where no charge transfer across the junction is allowed. The instantaneous energy levels, plotted versus time, form a set of parabolas, shown schematically in **Figure 2**. If the capacitor C is initially charged, a shifted set of parabolas is obtained, with the shift depending on the initial (discrete) charge on C. These sets mutually intersect. Introduction of a tunneling Hamiltonian removes some of the degeneracies of the intersecting parabolas. To first order in the tunneling Hamiltonian, a gap appears only between those parabolas that describe many-particle states connected by a single electron tunneling. This description assumes that a single value of the capacitance C is associated with all the microscopic states of the system.

*E. Shimshoni et al., unpublished; A. J. Leggett, Physics Dept., University of Illinois, Urbana, IL 61801, unpublished notes.

In the following discussion we are interested in the energy modulations of the system (in time) due to charging effects. The characteristic energy scale here is $e^2/8C$, which is much smaller than the thermal fluctuations $\sqrt{N}\,kT$. It is thus necessary to carry any measurement over a relatively long period of time in order to obtain a good signal-to-noise ratio. If we Fourier-transform the instantaneous energy of the system (assuming the adiabatic limit), in order to obtain a signal at frequency I_{ex}/e which is above the noise level, we need a measurement time t_m which satisfies

$$t_m > N(k_B T)^2(8C/e^2)^2(e/I_{ex})^2 t_{eq}^{-1}, \tag{3}$$

where t_{eq} is the equilibration time (the characteristic time scale over which the noise–noise correlation function vanishes). For $N = 10^9$, $kT = e^2/8C$, $e/I_{ex} \sim 10^{-10}$ s, $t_{eq} \sim 10^{-11}$ s, we obtain $t_m \gtrsim 1$ s.

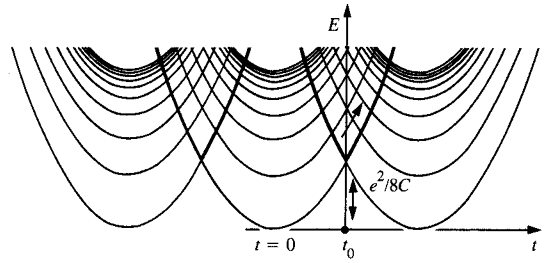

Figure 2

Shifted sets of energy levels corresponding to different initial charge on C (see text).

3. The coherent limit

We follow a wave packet in the action (energy–time) space (see Figure 2) as it evolves in time, while driving the system with external current I_{ex}. As we have noted in the previous paragraph, the system is initially described as a wave packet of energy width $\sqrt{N}\,kT$. When the inelastic rate is small, this wave packet may be considered to be coherent over some time interval. We have written the equations of motion that govern the quantum-mechanical evolution of the system in the action space, assuming the continuum limit. This is allowed when the energy level spacings ΔE are small and the current is large enough so that the probability NOT to undergo Zener tunneling between any pair of consecutive energy levels $1 - Pz$ is small and satisfies

$$1 - P_z = (\tilde{\Delta}E)^2 \frac{2\pi C}{\hbar e I_{ex}}, \tag{4}$$

where $\tilde{\Delta}E$ are the narrow energy gaps through which Zener tunneling may take place. We then have calculated the Green functions of this process; details of this calculation are given elsewhere [17]. The main result of our calculation is summarized below. Let us first note that in our picture, following the energy levels of one particular set of parabolas (and consequently being driven to increasingly higher energies) implies no charge transfer in real space. As we have discussed above, upon the introduction of small tunneling, Hamiltonian gaps open between previously intersecting parabolas. Hence, following these parabolas (see the arrow in Figure 2) implies that Zener tunneling through the gaps is very efficient (and the probability of charge tunneling in real space is small). Indeed, this is what we find in our analysis. One might think that even if the probability of not undergoing Zener tunneling through any given energy gap is small, say $1 - P_z = \varepsilon$ as we follow a given parabola through $1/\varepsilon$ gaps, the probability of not undergoing Zener tunneling at least once (and thus making a transition to the next set of parabolas, i.e., transferring charge across the junction) becomes significant. However, we find that this is NOT the case. As long as the coherent picture is valid, it is difficult for the system to reduce its energy by transferring charge to the other electrode. This is qualitatively different from the incoherent picture discussed in previous works [5, 7–8, 13–16].

4. The incoherent limit

Inelastic processes facilitate the relaxation of the system. In the presence of such processes, we expect the behavior discussed above to break down for large t.

In the coherent regime the wave packet under consideration follows the parabolic energy levels "ballistically." The rate at which the energy of the system increases is $dE/dt \equiv p \approx eI_{ex}/2C$ (this expression is obtained by linearizing the parabolas near the midpoint of intersection). On time scales larger than the phase-breaking time t_ϕ (but smaller than $t_{eq} \gtrsim t_\phi$), the wave packet broadens diffusively in the energy direction. Its energy width after time t is $t_\phi \cdot p(t/t_\phi)^{1/2}$. This width becomes of the order of the energy modulations $e^2/8C$ after time

$$\tau = \frac{1}{16}\left(\frac{e}{I_{ex}}\right)^2 t_\phi. \tag{5}$$

We thus expect the charge oscillations (of frequency I_{ex}/e) to have a finite lifetime $\sim \tau$ which gives rise to a finite spectral width.

Finally, we define the ohmic resistance R of the system by comparing the rate of dissipation $(I_{ex})^2R$ with the rate at which we pump energy into the system. The latter is estimated by noting that at each period ($t_p = e/I_{ex}$) the energy of the system is increased by $t_\phi \cdot p$ (for time $t < t_p$ the energy of the system increases roughly linearly in time. For times $t_\phi < t < t_p$, the wave packet broadens diffusively but the average energy does not increase). Therefore, the rate of pumping energy into the system is $pt_\phi/(e/I_{ex})$. We obtain $R = pt_\phi/eI_{ex}$, which implies

$$\tau = \frac{1}{32RC}\left(\frac{e}{I_{ex}}\right)^2. \tag{6}$$

For $I_{ex} = 10^{-9}$ A, $t_\phi = 10^{-12}$ s, and $C = 10^{-16}$ F, we find $R \approx 10$ kΩ, $t_p \approx 1.6 \cdot 10^{-10}$ s, and $\tau \simeq 10^{-9}$ s.

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