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Optimization Applied to the Design of an Energy-Efficient Building

Abstract: There are several public domain, and numerous proprietary, computer programs that provide detailed simulations of the heating and cooling requirements for a building. Such programs are often used to evaluate changes in the design of a building that are made to decrease its energy requirements. A user is considered to be working in a trial-and-error mode if each execution of the program provides no formal guidance for the next change. This work reports on an investigation of the imbedding of such an energy analysis program into an optimization structure. Such an arrangement would enable a user to specify a set of architectural and construction parameters and the limits within which they might vary, and from this to determine the parameters that yield a local minimum in thermal load and the sensitivity of this load to changes in these parameters.

Introduction

There are several public domain computer programs that are used both to calculate the heating and cooling energy (henceforth called "load") required to maintain the interior of a building within some prescribed temperature interval and to simulate the operation of equipment to supply that energy. [The National Bureau of Standards Load Determination Program, NBSLD (in combination with the Post Office Systems Simulation routine) is one, and the NASA Energy Cost Analysis Program, NECAP, is another.] If these programs are used to aid in the design of a new structure, or to assist in evaluating the efficacy of some energy-saving retrofit, the user is involved in a trial-and-error procedure with a very costly piece of software.

The central objective of the work described here has been to study the imbedding of these energy analyses into an optimization structure. If the user designates from among all the input data the particular architectural and construction parameters that may be varied and the allowable range of variation for each, an optimization algorithm can yield a sequence of decreasing heating and cooling loads and the building parameter values that result in those loads. In addition, the partial derivatives of load with respect to the parameters reveal the variation in the sensitivity of this load to changes in those parameters. The change in relative sensitivities as the design matures could offer useful information both for prescribing a mode of operation for the building and for future design work.

The first part of the paper discusses some of the benefits that might accrue from imbedding the load calculations into an optimization problem. The second portion presents the mathematical formulation of the constrained optimization problem and describes its implementation in an experimental program. The third part presents some preliminary numerical results from the optimization when the parameters are restricted to those governing heat conduction. The final section describes the optimization of both the heating and cooling load and an equipment-independent version of the systems load; in this form, the "load" minimized is the equivalent BTU cost arising from both parts.

Utility of optimization

The computer programs under discussion represent an attempt to use the increasingly available high-speed computer to yield accurate estimates of energy cost for a building as a function of time. The proliferation of users—voluntarily and/or through government urging, as with the State of California and Energy Conservation Regulations—obviously reflects concern over our energy supply and its cost.

The addition of the optimization methodology to NBSLD, or its equivalent, would considerably enhance its applicability and efficiency in design work. First, the answers to problems of sufficient complexity can be surprising—that is, they can be far from what intuition suggests. For example, minimization of energy load by optimizing solely over heat conduction parameters might yield unexpected results concerning the placement of layers of insulation and its effectiveness as a function of season. In addition, relaxing the tolerances on insulation properties gives us a new and solid tool for materials research. Second, attempting to decrease energy load by using NBSLD in a trial-and-error mode will probably not yield a sequence of monotonically decreasing answers, whereas with optimization it will; furthermore, one does not have to converge to a minimum to benefit from an optimization analysis. Third, the partial derivatives of energy load with respect to the optimizing parameters give us a measure of sensitivity, something quite useful but difficult to come by otherwise.

Although initially NBSLD (or a comparable program) combined with optimization would require more input data than the presently ample amount, and each run would be more time consuming, it seems that the potential benefits might warrant these drawbacks. Moreover, work is already in progress both to speed up the program and to decrease its input data requirements; for example, reference [1] discusses a novel filtering technique for significantly decreasing the amount of weather data needed to get an accurate estimate of a year's heating and cooling cost. Therefore, we consider the investigation reported herein to be both reasonable and timely.

Formulation

Several equations presented but not derived in this paper, relating to the radiation, convection, and conduction heat balance formulas, can be found in [2]. A glossary largely extracted from this reference follows.

Glossary

a	fraction of radiation absorbed in a single pass through a sheet of glass
c	coefficient of specific heat at constant pressure
CR	common ratio for the conduction transfer functions X, Y, Z
DB_t	outdoor air temperature at time t
$FO_{k,i}$	outside surface heat transfer coefficient
$F_{k,i}$	radiation heat exchange view factor between the ith and kth surface
GL	mass air flow rate due to air leakage
H_i	inside surface convection heat transfer coefficient for ith surface
IT	total solar radiation intensity on an outside surface
$K_{m,t}$	$[(1-RE)QEQUP + (1-RO)QOCPS + (1-RL)QLITE] \div \sum_{i=1}^{NS} S_i$
NS	number of surfaces contributing to the room heat balance
$Q_{m,t}$	heat conducted into mth inside surface at time t
$QEQUP$	internal heat generated from equipment
$QLITE$	heat from lights
QLS	sensible heat load
QO_t	heat loss at the exterior surface to the outdoor environment at time t
$QOCPS$	sensible internal heat generated from occupants
QR	incident solar radiation on an exterior surface
QS	heat loss to the sky, from an exterior surface
$R_{m,t}$	radiant heat flux impinging upon mth surface at time t
RE	fraction of internal heat gain from equipment that can be assumed to be convective
RL	fraction of internal heat gain from lights that can be assumed to be convective
RO	fraction of internal heat gain from occupants that can be assumed to be convective
S_i	area of ith heat transfer surface
SC	shading coefficient
SHG	solar heat gain through windows
$SUM1$	$\sum_{j=0}^{n} Y_j(TIS_{t-j} - TM) + CR(QO_{t-1})$
$SUM2$	$\sum_{j=1}^{n} Z_j(TOS_{t-j} - TM)$
TA_t	air temperature of the room at time t
$TIS_{i,t}$	inside temperature of surface i at time t
TM	a reference temperature
$TOS_{m,t}$	outside temperature of surface m at time t

In this first general setting, we formulate a constrained optimization problem the solution to which will minimize the total heating and cooling cost; this cost C is equivalent to that which would be computed by a "loads" program. We wish to minimize

$$C = N \sum_t |Q_h(t, w_j)| + \sum_t Q_c(t, w_j), \tag{1}$$

where t = time, N = ratio of heating cost to cooling cost, w_j = variables subject to optimization, and

$$Q_h(t, w_j) = \sum_{z \in Z_1(t)} q_h(t, z, w_j); \tag{2}$$

$$Q_c(t, w_j) = \sum_{z \in Z_2(t)} q_c(t, z, w_j). \tag{3}$$

The variable q_h denotes the heating (BTUs) required by a particular zone at a particular time, and Q_h is the sum of such loads over the set of zones that require heating; this set is denoted by Z_1, and is a function of time; q_c and Q_c

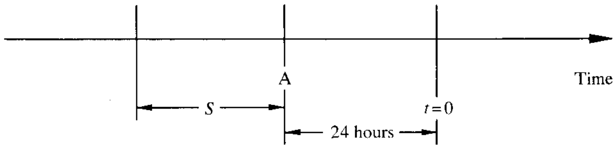

Figure 1 Time line depicting modified NBSLD initialization procedure; S is the time interval of steady state calculations, A is the time point at which calculations begin, and $t = 0$ is the starting time point for the problem.

are the similarly defined cooling parameters. The absolute value in Eq. (1) reflects the fact that load programs customarily compute the heating load as a negative number. We attempt to find values of the variables contained in the set $\{w_j\}$ which, when taken together, minimize the total heating and cooling cost. We are presently limiting these values to the following: wall conduction parameters k (thermal conductivity), ρc (volumetric specific heat), and L (thickness); emissivity E; solar absorption coefficient at an outside surface a; and shading coefficient SC. The constraints that can be imposed on the w_j are of the form

$$w_j^a \le w_j \le w_j^b;$$

$$e_{rj} w_j \ge d_r, \qquad r; j = 1, 2, \cdots, J; 1, 2, \cdots, J, \tag{4}$$

where w_j^a, w_j^b, e_{rj}, and d_r are all constants.

The constrained optimization problem is to minimize Eq. (1) subject to constraints of the form of Eq. (4), where the parameters w_j can be any or all of those in the set listed above.

Any constrained optimization procedure used to solve this problem is more accurate and executes faster if the first partial derivatives of C with respect to the w_j are in closed form:

$$\frac{\partial C}{\partial w_K} = N \sum_t \frac{Q_\mathrm{h}(t, w_j)}{|Q_\mathrm{h}(t, w_j)|} \left[\frac{\partial Q_\mathrm{h}}{\partial w_K}(t, w_j)\right] + \sum_t \frac{\partial Q_\mathrm{c}}{\partial w_K}(t, w_j), \qquad Q_\mathrm{h} \ne 0, \tag{5}$$

where w_K is any element of the set $\{w_j\}$.

In the notation of Reference [2], which we use whenever possible,

$$QLS \equiv Q_\mathrm{h}, \qquad \text{if } QLS < 0;$$

$$QLS \equiv Q_\mathrm{c}, \qquad \text{if } QLS \ge 0.$$

To emphasize the point that a summation over zones is involved, we write

$$Q_\mathrm{h} = \sum_{z\in Z_1} QLS(t, z, w_j), \qquad QLS < 0 \text{ for } z \in Z_1;$$

$$Q_\mathrm{c} = \sum_{z\in Z_2} QLS(t, z, w_j), \qquad QLS \ge 0 \text{ for } z \in Z_2. \tag{6}$$

From Appendix A of [2] we find that

$$QLS(t, z, w_j) = \sum_{i=1}^{NS} H_i S_i (TIS_{i,t} - TA_t) + GL_t c(DB_t - TA_t) + (QEQUP)RE + (QOCPS)RO + (QLITE)RL, \tag{7}$$

and

$$\frac{\partial QLS}{\partial w_K}(t, z, w_j) = \sum_{i=1}^{NS} H_i S_i \frac{\partial TIS_{i,t}}{\partial w_K}. \tag{8}$$

Now $TIS_{i,t}$ is found by solving a set of simultaneous equations

$$A_{mi} TIS_{i,t} = B_m', \qquad m = 1, 2, \cdots, NS, \tag{9}$$

where the matrices A_{mi} and B_m' are fully described in [2, 3]. We assume in this formulation that the air temperature is prescribed. Differentiating (9) we find

$$\frac{\partial TIS_{i,t}}{\partial w_K} = A_{mi}^{-1}\left(\frac{\partial B_m'}{\partial w_K} - \frac{\partial A_{mi}}{\partial w_K} TIS_{i,t}\right), \tag{10}$$

where A_{mi}^{-1} is the inverse of the matrix A_{mi}, and is known from the solution of Eq. (9).

The partial differentiation of B_m' and A_{mi} is extensive though straightforward, and may be found detailed in [4].

In the course of the solution for the sensible heat load QLS, all inside and outside surface temperatures (TIS, TOS) are determined, as are all inside and outside wall heat fluxes (Q, QO). The calculation of the partial derivatives of QLS requires initial values for the partial derivatives of these temperatures and fluxes, and their determination is now discussed.

The initialization procedure used with the author's version of NBSLD is more extensive than the nominal form. The modification was done to provide greater accuracy in load and less sensitivity to initial conditions for calculations over time intervals on the order of weeks. Specifically, the first day of weather data is assumed to exist one day back in time from the starting day, and a certain number of hours of constant indoor and outdoor temperatures are assumed to prevail prior to that interval; the particular number is equal to the maximum of the number of response factors in all the exterior walls. Therefore, a steady state solution is given for the interval S, shown in Fig. 1, and calculations in the program begin at point A. All loads computed for times prior to $t = 0$ are dropped. The optimization requires partial derivatives for times prior to the starting point, but from the above description this means only over the interval S. The partial derivatives of TIS, TOS, and the boundary flux for an arbitrary composite wall are computed as follows.

Let the thicknesses and conductivities of the layers be denoted by L_i and k_i, respectively, where i runs from 1 to

Table 1 Initial values of heat conduction parameters for wall layers shown in Fig. 2.

Layer no.	L (ft; m)	k (BTU/hr-ft-°F; J/s-m-K)	ρc (BTU/ft³-°F; 10^5J/m³-K)	*Resistivity* (ft²-hr-°F/BTU; m²-s-K/J)
1	0.0312; 0.0095	0.12; 0.54	14.24; 9.68	—
2	0.15; 0.046	0.027; 0.047	1.14; 0.78	—
3	0.06; 0.019	—	—	0.2; 0.035
4	0.15; 0.046	0.027; 0.047	1.14; 0.78	—
5	0.28; 0.085	0.58; 1.00	25.0; 17.0	—

NL, NL being the number of layers. The total resistance of the wall is computed as

$$R = \sum_{i=1}^{NL} \frac{L_i}{k_i} + \sum_{j=1}^{NL} \frac{1}{CN_j},$$

where the first summation represents the material layers, the second summation, the air layers; the CN_j are the conductances of the air layers. The wall U factor is

$$U = 1/R.$$

The equation for one-dimensional steady state heat conduction in a homogeneous wall is

$$\partial^2 T/\partial x^2 = 0,$$

from which the solution for temperature T is

$$T = \alpha + \beta x. \tag{11}$$

Therefore, the required derivatives are evaluated by using a hypothetical single-layer wall with the same total resistance as the actual composite wall.

The boundary conditions are on the flux q, and are

$$q_i = -UL \frac{\partial T}{\partial x}\bigg|_{x=L} = h_i[T(L) - TA],$$

and

$$q_0 = -UL \frac{\partial T}{\partial x}\bigg|_{x=0} = h_0[DB - T(0)], \tag{12}$$

where h_i and h_0 are the surface heat transfer coefficients for the inside and outside layers, respectively, TA is the inside air temperature, and DB is the outside dry bulb temperature. Using Eqs. (11) and (12), we find

$$\alpha = \frac{h_0 DB(1 + h_i R) + h_i TA}{h_i(1 + h_0 R) + h_0}$$

$$\beta = h_0(\alpha - DB)/UL. \tag{13}$$

The partial derivatives needed are

$$\frac{\partial TIS}{\partial w_K} = \frac{\partial T}{\partial w_K}\bigg|_{x=L} = \frac{\partial \alpha}{\partial w_K} + L\frac{\partial \beta}{\partial w_K},$$

$$\frac{\partial TOS}{\partial w_K} = \frac{\partial T}{\partial w_K}\bigg|_{x=0} = \frac{\partial \alpha}{\partial w_K},$$

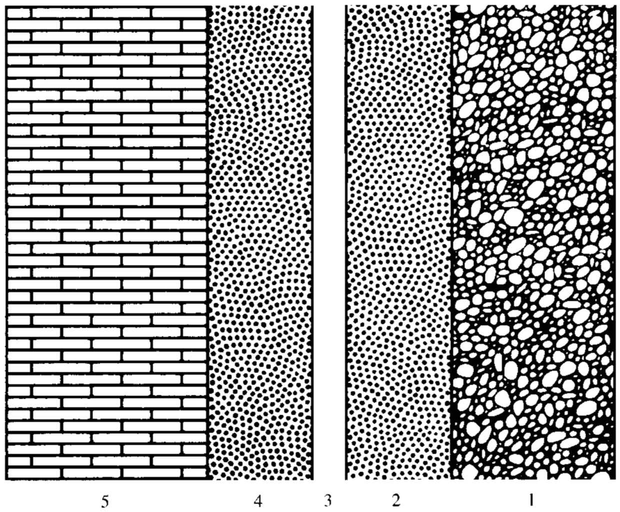

Figure 2 Wall cross section. Layers are denoted as follows: 1, gypsum board; 2, insulation; 3, air layer; 4, insulation; 5, brick.

and

$$\frac{\partial}{\partial w_K}\left(-k\frac{\partial T}{\partial x}\bigg|_{x=0,L}\right) = -UL\frac{\partial \beta}{\partial w_K} - \beta\frac{\partial (UL)}{\partial w_K}.$$

These are readily found from Eqs. (12) and (13).

Numerical results

The methodology formulated in the previous section has been implemented on a problem for which the parameter set $\{w_j\}$ involves only the heat conduction portion of the loads calculation. Specifically, our test case used the single-zone Fort Myer building (see Appendix D of [2]) with the exterior wall composition modified to allow for two layers of insulation. Figure 2 depicts a cross section of the wall, and Table 1 lists the starting values of the wall properties. Our first problem optimized on k and ρc in layers 2 and 4; the thickness of each layer was fixed at 0.05 m (0.15 ft), as shown. The constraints imposed were as follows:

$$0.01 \le k_j \le 2.0, \quad j = 2, 4 \ (k_j \text{ in units of BTU/ft-hr-°F});$$

$$0.5 \le \rho c_j \le 40.0 \qquad (\rho c_j \text{ in units of BTU/ft}^3\text{-°F}).$$

Internal heat loads included light and occupants, and the schedules were slightly modified from those in [2]. In

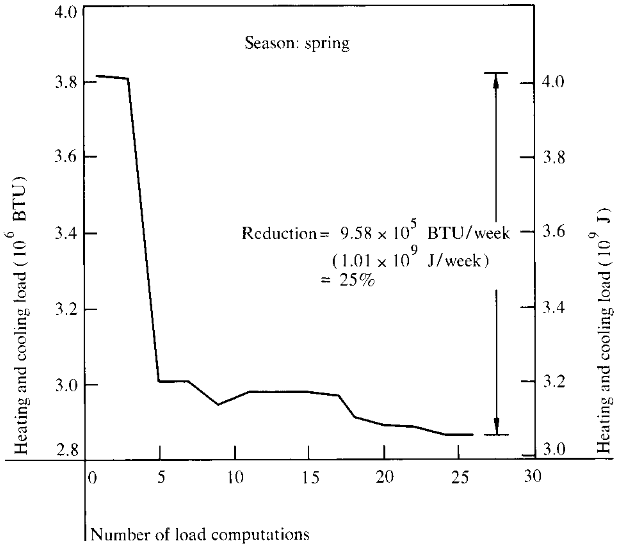

Figure 3 Optimization of thermal load with respect to conductivity and volumetric specific heat for insulation layers 2 and 4 (spring season).

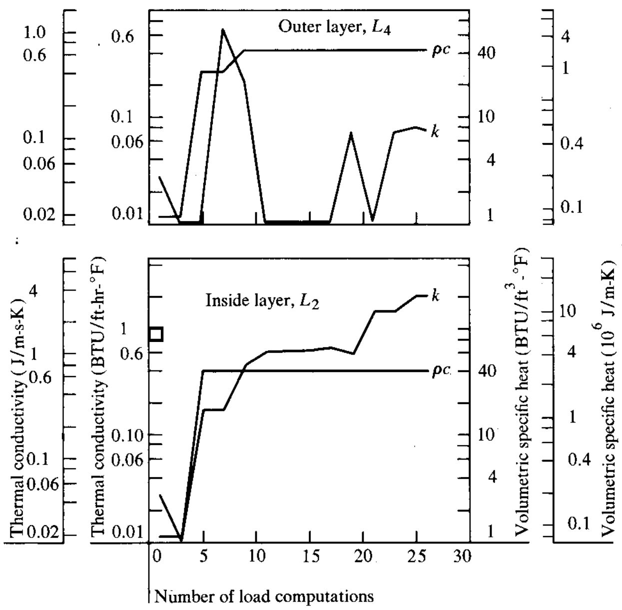

Figure 4 Variations in thermal conductivity k and volumetric specific heat ρc as optimization progresses (spring season).

addition, the building had 0.635 cm (1/4 in.) plate glass windows of 9.3 m^2 (100 ft^2) on the 23.3 m^2 (250 ft^2) wall, and 29.8 m^2 (320 ft^2) on the 54.0 m^2 (900 ft^2) wall.

Some results in [1] have made it possible to construct a small set of weather data which, when used with any energy analysis program, yields heating and cooling loads that are very similar to those obtained by using a full year of weather data with the same program. This result greatly enhances the practical application of optimization, since such problems require repeated calculation of the cost function. Because of this finding, and the desire to minimize computer time, the optimization problem was solved with only one week of weather data for each of the winter, spring, and summer seasons of 1973 Los Angeles weather. The constrained optimization itself was implemented with subroutine VE01A from the Harwell Subroutine Library [5]; this subroutine uses the variable metric method.

The results shown in Figs. 3-5 and Table 2 are only for the single week of spring data. The intent is simply to demonstrate the feasibility of the technique; no claim is made as to the suitability of these results on an annual basis. Figure 3 illustrates the decrease in the heating and cooling load as the optimization progresses. The abscissa does not represent iterations in the optimization sequence, but the more frequent calculations of load. Table 2 shows that the ratio of heating to cooling requirements was virtually the same at the end of the optimization as it was at the beginning, although, as seen in Fig. 3, a reduction of about 25 percent had been achieved in their sum. Figure 4 displays the change in k and ρc in the inner and outer layers of insulation.

One's intuition might suggest that the lower the conductivity of the insulation, the lower would be the building's energy requirements. This is clearly not the case in this example, and it is possibly not true in general. The intuitive statement perhaps accurately reflects savings with conventional materials, but the results demonstrate that different materials might yield substantial savings. In this example, the high conductivity and heat capacity of the inside insulation layer suggest that the internal heat loads in the building and the thermal storage of the walls are in greater harmony now than with the starting handbook values.

This result also suggests a use for such computer programs other than building energy simulation. For example, they can be used for building materials research. If one selects a set of generic building types, each representative of a class, and a set of weather regions, such optimization methods may be used to find optimal insulation materials, glass coatings, interior wall compositions, and so on.

The spring weather example was pushed one step further by using the optimal values of k and ρc in layers 2 and 4 (insulation), and by solving for the optimal thicknesses of those layers. The constraints required that

$$L_2 + L_4 = 0.30 \qquad \text{(in ft)};$$

$$L_j \geq 0, \qquad j = 2, 4.$$

As can be seen in Fig. 5, the first departure from initial conditions was to use the available length for the most efficient layer of insulation (as found from $C/\partial L_j$), which is an intuitively clear way to proceed. However, this caused a considerable increase in load, and the optimization proceeded to retain both layers, finally settling on twice as much insulation in the inside layer adjacent to the gypsum board as in the outside layer adjacent to the brick.

Preliminary integration of equipment and thermal load

We now present an extension of the previous formulation, one which integrates the system equipment load with the heating and cooling load. Our approach is only a first step in that direction, but it is one that satisfies an important precondition. We wish to include the system load in our objective function, but not at the expense of having to select a specific air handling system, e.g., dual duct, variable air volume, etc. That is, we would like the output of the optimization to be the basis of the problem for an air conditioning engineer. To this end, we have described an idealized internal heat pump operation, as shown in Fig. 6; the cooling zone represents the collection of all zones requiring cooling at one point in time, and the heating zone represents the collection of all zones requiring heating at that same point in time.

We assume that the coefficient of performance (COP) of the heat pump is specified, and that we can relate it to the loads by

$$COP = \min\,(Q_c + M, |Q_h|)/M,$$

where

$$Q_h(t, w_j) = \sum_{z \in Z_1} QLS(t, z, w_j),$$

$$Q_c(t, w_j) = \sum_{z \in Z_2} QLS(t, z, w_j),$$

and M equals the motor load or the "BTU cost" to operate the heat pump, and is a function of time, i.e., $M(t)$. We wish to minimize

$$\Lambda = N_1 \sum_{t \in T_1(t)} |Q(t, w_j)| + N_2 \sum_{t \in T_2(t)} Q(t, w_j) + N_3 \sum_t M(t), \qquad (14)$$

where

$$Q(t, w_j) = Q_h(t, w_j) + Q_c(t, w_j) + M(Q_h, Q_c),$$

N_1, N_2, N_3 are constants, and

$t \in T_1(t)$ if $Q(t, w_j) < 0$;

$t \in T_2(t)$ if $Q(t, w_j) \geq 0$.

Therefore, we are minimizing Λ, the net heating and cooling load plus M.

The Heaviside step function is

$$H(x) = \begin{bmatrix} 1, & x > 0 \\ 0, & x \leq 0 \end{bmatrix}.$$

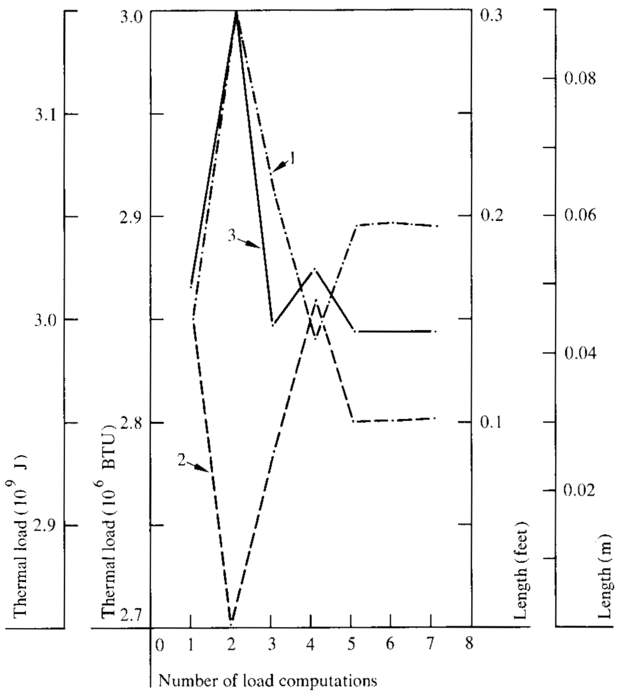

Figure 5 Thermal load and history of optimized insulation thicknesses, where curves 1 and 2 are the thicknesses of layers 2 and 4, respectively, and curve 3 is the thermal load (spring season).

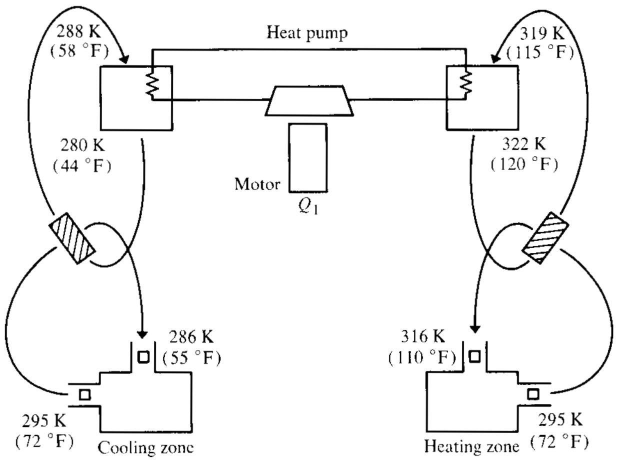

Figure 6 Representative internal heat pump configuration (note: temperatures shown are for illustrative purposes only).

Table 2 Total heating and cooling loads at the beginning and end of optimization.

	Heating (10^6 BTU/wk; 10^9 J/wk)	*Cooling* (10^6 BTU/wk; 10^9 J/wk)	*Ratio*
Start	1.601; 1.69	2.223; 2.35	0.720
End	1.202; 1.27	1.664; 1.76	0.722

We can write Eq. (14) as

$$\Lambda = N_1 \sum_t [1 - H(Q)] |Q(t, w_j)| + N_2 \sum_t H(Q)Q(t, w_j) + N_3 \sum_t M(t). \tag{15}$$

Now,

$$M = \min (|Q_h|, Q_c + M)/COP; \tag{16}$$

letting $v = (M + Q_c)/(|Q_h| + \varepsilon)$, we can again use $H(x)$ to express (16) as

$$M = H(v - 1) \frac{|Q_h|}{COP} + [1 - H(v - 1)] \frac{Q_c}{COP - 1}. \tag{17}$$

The small quantity ε is included in the definition of v to ensure that if $Q_h = 0$, $H(v - 1) = 1$. Combining Eqs. (15) and (17), we have the minimizing function in the case of an idealized internal heat pump operation,

$$\Lambda = N_1 \sum_t [1 - H(Q)]|Q(t, w_j)| + N_2 \sum_t H(Q)Q(t, w_j) + N_3 \sum_t \left\{H(v - 1) \frac{|Q_h|}{COP} + [1 - H(v - 1)] \frac{Q_c}{COP - 1}\right\} + \frac{N_3}{2} \sum_{t \in T_3} \frac{|Q_h|}{COP} + \frac{Q_c}{COP - 1},$$

where T_3 is the set of time points where $v = 1$.

To compute the first partial derivative of Λ with respect to a w_K, first replace $H(x)$ with a function that is an arbitrarily close approximation of $H(x)$:

$$H_n(x) = \sqrt{\frac{n}{\pi}} \int_{-\infty}^{x} \exp(-ny^2)dy.$$

This will avoid indeterminacy in $\partial H/\partial Q$ when $Q = 0$. We have stipulated in Eq. (14) that the set of time points for which $Q = 0$ is contained in the set T_2 (cooling loads);

$$\begin{aligned}\frac{\partial \Lambda}{\partial w_K} &= N_1 \sum_t \left[\frac{-\partial H_n}{\partial Q} (Q) \frac{\partial Q}{\partial w_K}\right] |Q(t, w_j)| \\ &+ N_1 \sum_t [1 - H_n(Q)] \frac{Q(t, w_j)}{|Q(t, w_j)|} \frac{\partial Q}{\partial w_K} (t, w_j) \\ &+ N_2 \sum_t \frac{\partial H_n}{\partial Q} (Q) \frac{\partial Q}{\partial w_K} Q(t, w_j) \\ &+ N_2 \sum_t H_n(Q) \frac{\partial Q}{\partial w_K} (t, w_j) \\ &+ N_3 \sum_t \left\{\left[\frac{\partial H(v - 1)}{\partial (v - 1)}\right] \left[\frac{\partial (v - 1)}{\partial w_K}\right] \frac{|Q_h|}{COP}\right. \\ &+ \left[\frac{H(v - 1)}{COP}\right] \left[\frac{Q_h}{|Q_h|}\right] \frac{\partial Q_h}{\partial w_K} (t, w_j) \\ &- \left[\frac{\partial H(v - 1)}{\partial (v - 1)}\right] \left[\frac{\partial (v - 1)}{\partial w_K}\right] \frac{Q_c}{COP - 1} \\ &\left.+ \left[\frac{1 - H(v - 1)}{COP - 1}\right] \frac{\partial Q_c}{\partial w_K}\right\} \\ &+ \frac{N_3}{2} \sum_{t \in T_3} \left[\frac{Q_h}{|Q_h|} \frac{\partial Q_h}{\partial w_K} \frac{1}{COP} + \left(\frac{1}{COP - 1}\right) \frac{\partial Q_c}{\partial w_K}\right].\end{aligned} \tag{19}$$

The first sum approaches zero in the limit as $n \to \infty$. For the second sum, $H_n(Q) \to H(Q)$ in the limit as $n \to \infty$. In the third sum, assuming finite $\partial Q/\partial w_K$, the product approaches zero as $n \to \infty$. The terms $[\partial H(v - 1)/\partial (v - 1)] = 0$; therefore Eq. (19) reduces to the following:

$$\begin{aligned}\frac{\partial \Lambda}{\partial w_K} &= N_1 \sum_t [1 - H(Q)] \left[\frac{Q(t, w_j)}{|Q(t, w_j)|}\right] \frac{\partial Q}{\partial w_K} (t, w_j) \\ &+ N_2 \sum_t H(Q) \frac{\partial Q}{\partial w_K} (t, w_j) \\ &+ N_3 \sum_t \left\{\left[\frac{- H(v - 1)}{COP}\right] \frac{\partial Q_h}{\partial w_K} (t, w_j) + \left[\frac{1 - H(v - 1)}{COP - 1}\right] \frac{\partial Q_c}{\partial w_K}\right\} \\ &+ \frac{N_3}{2} \sum_{t \in T_3} \left[\left(\frac{-1}{COP}\right) \frac{\partial Q_h}{\partial w_K} + \left(\frac{1}{COP - 1}\right) \frac{\partial Q_c}{\partial w_K}\right].\end{aligned} \tag{20}$$

Since $Q = Q_h + Q_c + M$, and M is given in terms of Q_h and Q_c from Eq. (17), Eq. (20) takes a form equivalent to that of Eq. (5) in the second section. Equation (20), together with the results of the second section, yields an implementable simulation of the internal heat pump total load. It is believed that this formulation represents a meaningful first order integration of the (unspecified) systems' equipment load with the heating and cooling load. In particular, that part of the output consisting of the set of zones Z_1 and Z_2 as a function of time is a guide to the mechanical engineer's design and selection of equipment. In addition, the minimum load derived by the program provides a benchmark against which alternative designs can be measured.

Conclusions

The technique of optimization has been applied to building energy analysis programs and has shown promise of being a cost-effective method in building design. Trial-and-error methods rooted in intuition can be replaced by optimization methods and the possibility of the non-intuitive solution. This is particularly true when this method is applied to the heat pump problem, for the user can determine the effect of alternative spatial allocations of people, laboratories and other facilities on the dynamic interconnection of zones, feasibility of implementation, and minimum energy load. Although the results herein have demonstrated cost savings only with the conduction aspect, it is clear that the technique can be extended to a wide range of architectural and construction parameters.

The practical application of the optimization technique can best be explained by referring back to the sample problem in the section on numerical results: a) Set the constraints on k and ρc slightly beyond the limits of available materials; b) find the optimal values of k and ρc for the two layers of insulation; c) pick actual insulations whose properties are as close as possible to the optimum ones; and d) by using the properties k and ρc of the available materials, solve for the optimal thicknesses.

The section on numerical results has also demonstrated the attractiveness of using this technique in building materials research, where the user exploits the "in vivo" nature of the simulation; that is, the type of building, the climate to which it will be subjected, and its use are taken into account in the search for energy-efficient building materials.

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