F. Partovi

Note on Perturbation of a Uniform Magnetic Field by a Cube of Magnetic Material

The following analysis deals with the problem of determining the magnetic field in the vicinity of a rectangular parallelepiped ($2b \times 2a \times 2a$) of high permeability μ placed in a uniform magnetic field H_0 as shown in Fig. 1. The solution is to be obtained through successive approximations. Numerical results from the first approximation are compared with experimental data.

The magnetic scalar potential V' at a point (X, Y, Z) is given by[1]

$$V'(X, Y, Z) = -H_0 X + \frac{1}{4\pi\mu_0} \left(\int_s \frac{\mathbf{M}' \cdot \mathbf{ds}}{r} - \int_v \frac{\nabla \cdot \mathbf{M}'}{r} dv \right), \tag{1}$$

where μ_0 is the permeability of free space, $\mathbf{ds}$ is the normal vector directed outwards representing the element of surface area on the sample, dv is the element of volume within the sample, r is the distance from the point (X, Y, Z) to a point (x, y, z) of the parallelepiped, and $\mathbf{M}'$ is equal to the magnetization within the parallelepiped.

In the first order approximation, the rectangular parallelepiped is assumed to be uniformly magnetized in the X-direction with $\mathbf{M}'$ equal to $M'\mathbf{i}$, where $\mathbf{i}$ is a unit vector in the X-direction. Equation (1), after dividing by H_0, is reduced to

$$V_1(X, Y, Z) = -X + \frac{M}{4\pi\mu_0} \int_{-a}^{a} \int_{-a}^{a} \cdot \left[\frac{1}{(b-X)^2 + (y-Y)^2 + (z-Z)^2} - \frac{1}{\sqrt{(b+X)^2 + (y-Y)^2 + (z-Z)^2}} \right] dy dz, \tag{2}$$

where

$$V_1 = \frac{V'}{H_0}, \quad \text{and } M = \frac{M'}{H_0}.$$

From this point on, the subscripts 1, 2, etc., refer to the orders of approximation.

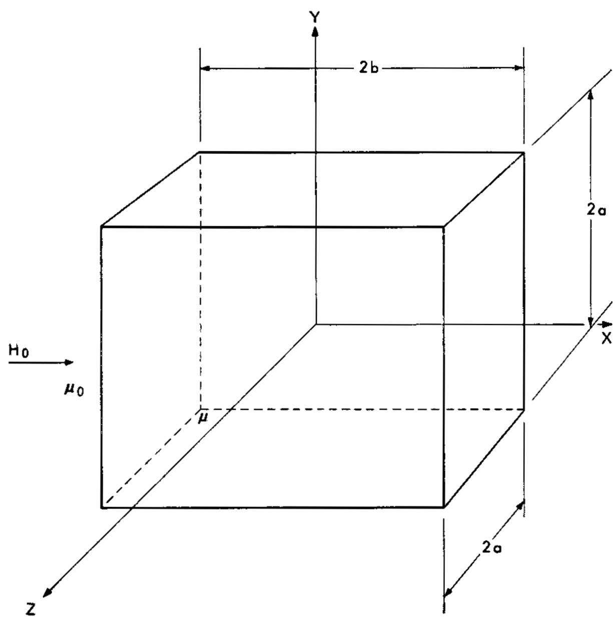

Figure 1 **Configuration of parallelepiped in magnetic field.**

Equation (2) is readily integrable and the result is

$$V_1(X, Y, Z) = -X + \frac{M}{4\pi\mu_0}\,[f(-X, Y, Z) - f(X, Y, Z)]\,, \tag{3}$$

where

$$\begin{aligned} f(X, Y, Z) = {} & \Gamma(X, Y, Z) + \Gamma(X, -Y, Z) + \Gamma(X, Y, -Z) + \Gamma(X, -Y, -Z) \\ & + \Gamma(X, Z, Y) + \Gamma(X, Z, -Y) + \Gamma(X, -Z, Y) + \Gamma(X, -Z, -Y) \\ & - (b+X)\Bigg[\tan^{-1}\frac{(b+X)^2+(a-Y)(a-Y+\sqrt{(b+X)^2+(a-Y)^2+(a+Z)^2})}{(b+X)(a+Z)} \\ & + \tan^{-1}\frac{(b+X)^2+(-a-Y)(-a-Y+\sqrt{(b+X)^2+(a+Y)^2+(a-Z)^2})}{(b+X)(-a+Z)} \\ & + \tan^{-1}\frac{(b+X)^2+(a-Y)(a-Y+\sqrt{(b+X)^2+(a-Y)^2+(a-Z)^2})}{(b+X)(a-Z)} \\ & + \tan^{-1}\frac{(b+X)^2+(-a-Y+\sqrt{(b+X)^2+(a+Y)^2+(a+Z)^2})}{(b+X)(-a-Z)}\Bigg], \end{aligned} \tag{4}$$

and

$$\Gamma(X, Y, Z) = (a+Z)\log\frac{a+Y+\sqrt{(b+X)^2+(a+Y)^2+(a+Z)^2}}{\sqrt{(b+X)^2+(a+Z)^2}}\,. \tag{5}$$

The X, Y, Z components of the magnetic field intensity H_1 can be obtained from

$$H_{1_x} = -\frac{\partial V_1}{\partial X}\,, \qquad \text{etc.} \tag{6}$$

The magnetization $\mathbf{M}_1$ can then be calculated from

$$\mathbf{M}_1 = (\mu - \mu_0)\mathbf{H}_1\,. \tag{7}$$

To obtain a second order approximation, one can use the magnetization $\mathbf{M}_1$ obtained in the first approximation and substitute it into Equation (1), from which V_2, $\mathbf{H}_2$ and M_2 can be calculated thus completing another cycle of approximation. The same process can be extended to any desirable higher order of approximation.

A bizmuth Hall probe[2] was used to measure the magnetic field outside the parallelepiped.

The volume of the parallelepiped is approximately one-sixtieth of the volume in which the field might be considered uniform and the effective area of the probe is 10^4 times smaller than that of one face of the parallelepiped. The experimental data were compared with the numerical results from the first approximation. The disagreement between the two is less than 5 per cent of the measured value everywhere except in the regions near the corners. Also, on the surface, $X=b$, $\mu_0 H_{1_x}(b+0, Y, Z)$ and $\mu H_{1_x}(b-0, Y, Z)$ are practically equal. Therefore, for all practical purposes, the first approximation is generally sufficient.

References

1. E. R. Peck, *Electricity and Magnetism,* McGraw-Hill Book Co., p. 290, 1953.
2. B. Kostyshyn and D. D. Roshon, Jr., *Proc. IRE* **47,** 451, (1959).

Revised manuscript received September 16, 1959